

Numerical Solution for the Velocity-Derivative Skewness of a Low-Reynolds-Number Decaying Navier-Stokes Flow

Robert G. Deissler
Lewis Research Center
Cleveland, Ohio

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NUMERICAL SOLUTION FOR THE VELOCITY-DERIVATIVE SKEWNESS OF A
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Robert G. Deissler
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135

Abstract

The variation of the velocity-derivative skewness of a Navier-Stokes flow as the Reynolds number goes toward zero is calculated numerically. The value of the skewness, which has been somewhat controversial, is shown to become small at low Reynolds numbers.

We write the skewness S of the velocity derivative $\partial u_1/\partial x_1$ as

$$S = \frac{\overline{\left(\partial u_1/\partial x_1\right)^3}}{\left[\overline{\left(\partial u_1/\partial x_1\right)^2}\right]^{3/2}} \quad (1)$$

where u_1 is the velocity component in the x_1 -direction and the overbars indicate averaged values. If the skewness (or the skewness factor) is a measure of the nonlinearity of the flow, one might expect it to approach zero as the Reynolds number becomes small, since the Navier-Stokes equations should be linear at low enough Reynolds numbers. Batchelor and Townsend [1] have, in fact, given a relation between the nonlinear term in the vorticity equation and the skewness.

Nevertheless, some plausible analyses have indicated that the skewness factor may not approach zero at vanishingly small Reynolds numbers. In particular, Refs. [2] and [3], where the shape of the energy spectrum was assumed to remain similar with increasing time, concluded that the skewness factor approaches a nonzero value in the final period of decay. That value did not differ greatly from those at earlier times. On the other hand an analysis where the assumption of similarity was not invoked [4 and 5], and an experiment [5], indicated that the velocity-derivative skewness approaches zero in the final period.

Here we use a numerical solution for a decaying Navier-Stokes flow to determine how the skewness changes with time. The Navier-Stokes equations for an incompressible flow can be written as

$$\frac{\partial u_i}{\partial t} = - \frac{\partial(u_i u_k)}{\partial x_k} - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_k \partial x_k}, \quad (2)$$

where the pressure is given by the Poisson equation

$$\frac{1}{\rho} \frac{\partial^2 p}{\partial x_l \partial x_l} = - \frac{\partial^2 (u_l u_k)}{\partial x_l \partial x_k}. \quad (3)$$

The subscripts can have the values 1, 2, or 3, and a repeated subscript in a term indicates a summation, with the subscript successively taking on the values 1, 2, and 3. The quantity u_i is an instantaneous velocity component, x_i is a space coordinate, t is the time, ρ is the density, ν is the kinematic viscosity, and p is the instantaneous pressure. As in [6] the initial velocity is given by

$$u_i = a_i \cos \mathbf{q} \cdot \mathbf{x} + b_i \cos \mathbf{r} \cdot \mathbf{x} + c_i \cos \mathbf{s} \cdot \mathbf{x}, \quad (4)$$

where

$$\begin{aligned} a_i &= k(2,1,1), & b_i &= k(1,2,1), & c_i &= k(1,1,2), \\ q_i &= (-1,1,1)/x_0, & r_i &= (1,-1,1)/x_0, & s_i &= (1,1,-1)/x_0, \end{aligned} \quad (5)$$

k is a quantity that fixes the initial Reynolds number at $t = 0$, and x_0 is one over the magnitude of an initial wavenumber component. The initial pressure is not specified since it is calculated from Eq. (3). Equations (4) and (5) satisfy continuity, and Eqs. (2) and (3) insure that continuity is maintained. The boundary conditions are periodic, with a period of $2\pi x_0$.

It has been shown in [7] that 128^3 grid points appear to resolve our flow, even for times when the Reynolds number is relatively high (initial Reynolds number $\sim 1,000$). Since we are mainly concerned here with the final period of decay, where the Reynolds number is low, the results of most interest should be very well resolved spatially. As in [6], numerical stability limitations force the timewise resolution to also be good.

Figure 1 shows the calculated time evolution of the velocity-derivative skewness. At the earlier times the skewness oscillates, having an average

value somewhere around 0.5. That type of behavior was also obtained for the numerical calculations in [8]. For later times S goes monotonically toward zero as the fluctuations decay.

The calculated variation of the skewness with the microscale Reynolds number $R_\lambda = \frac{\overline{u^2}}{\lambda/\nu}$ is plotted logarithmically in Fig. 2, where $\overline{u^2} = \overline{u_i u_i}/3$ and λ is the Taylor microscale. Except at the larger Reynolds numbers, the skewness goes toward zero monotonically as R_λ decreases. The limiting rate of approach to zero for the numerical solution is about the same as that for the experiment in [5] ($S \propto R_\lambda^{-1.4}$).

The nonsimilar analyses in [4] and [5] give skewness factors that approach zero as the Reynolds number decreases, as do the skewness factors in our numerical solution and the experiment in [5]. This is in contrast to similarity solutions [2 and 3]. The results suggest that the use of similarity assumptions for calculating skewness factors in the approach to the final period of decay may not always give realistic trends; the change in shape of the energy spectrum with time seems to have an important effect.

As a final observation we note that at early times the fluctuations in our flow are time-dependent and display sensitive dependence on initial conditions [7]. The flow at early times therefore contains important ingredients of turbulence [9]. That is evidently not the case at later times in the approach to the final period of decay. In [10] it is shown that, at least for forced flow, the fluctuations become time-independent at Reynolds numbers ~ 5 and below. The fluctuations are then completely spatial. Time-independent fluctuations also occur for our present flow in the approach to the final period of decay, where the Reynolds numbers are on the order of

one and less (see, for example, Fig. 3). Thus the flow in the approach to the final period cannot be considered truly turbulent. Batchelor and Townsend [11] recognized early the nonturbulent character of the flow in the final period of decay. Our results indicate that the flow in the *approach* to the final period is also not turbulent in a strict sense, since the fluctuations are time-independent, being only spatial. The presence of spatial fluctuations in the absence of temporal fluctuations is shown by the fact that u_1 is different at the two points considered in Fig. 3, even when temporal fluctuations are absent.

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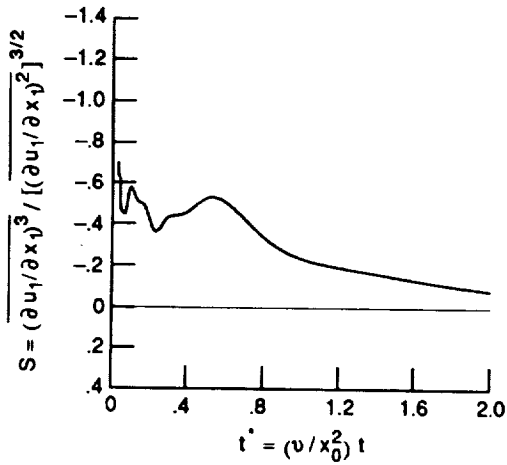


Figure 1.—Calculated time-evolution of velocity-derivative skewness for a decaying flow. Overbars designate space averages.

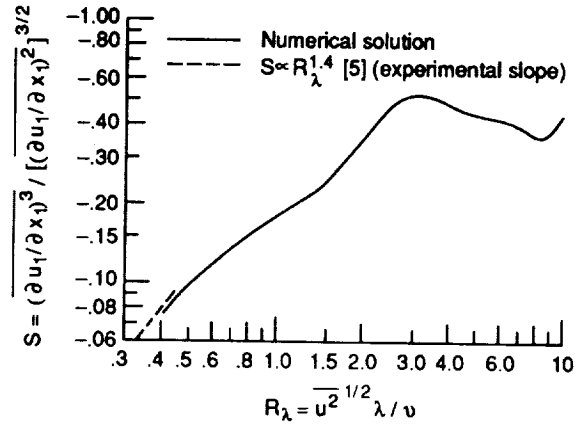


Figure 2.—Variation of velocity-derivative skewness with microscale Reynolds number.

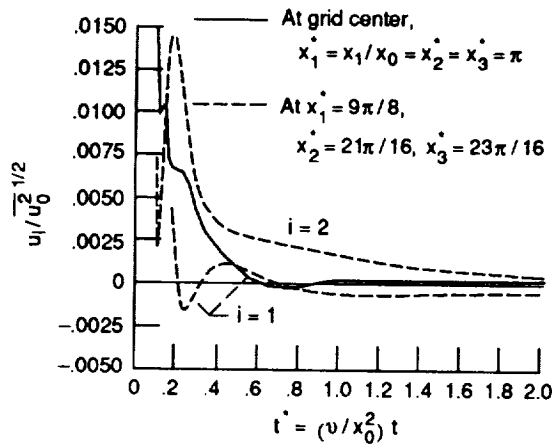


Figure 3.—Calculated time-evolution of un-averaged velocity components for a decaying flow. Ordinates normalized by initial condition.

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